[SP26] ECN 812B Recitation 1

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1 Concepts this Week

In studying strategic games, we assume that players are **rational** agents. Tangent to rationality, we will also assume

- Perfect Recall: Agents know what they did
- Common Knowledge: See lecture notes

Other important definitions are:

- Perfect Information: Every player knows the complete history of the game.
- Normal form game: List of players, strategies, and payoffs. Payoffs are often specified by a payoff table, but a table is not necessary.
- Extensive form game: List of players, nodes and mappings, information sets, and payoffs. Commonly, but not necessarily, represented by a game tree.

Note that all games can be represented in either form, but it's generally easier to represent simultaneous games with normal form, and sequential games with extensive form.

- Strategy: A *complete contingent plan* of a player. The strategy set is the Cartesian product of the set of actions from each information set.
  - Pure Strategy: A strategy s_i for player i is a pure strategy if at every node/information set, *only one action* is played (with probability 1).
  - Behavior Strategy: A strategy λ_i for player i is a behavior strategy if at some node/information set, more than one action is played with non-zero probability.
  - Mixed Strategy: A strategy σ_i for player i is a mixed strategy if some s_i is played with strictly positive probability.
- Strategy Profile: A vector of players' strategy (Not strategy set).
- (Weak) Dominance: For a given player, strategy s (weakly) dominates strategy s' if the payoff of playing s is always (weakly) higher than that of playing s', given the strategy profile of the other players.

- Iterative Elimination of Weakly/Strictly Dominated Strategies
  - IEWDS may yield different solutions given the order of elimination.
  - IESDS always yields the same solution(s) regardless of the order of elimination.
- Dominant Strategy Solution: The strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is a dominant strategy solution if $\forall i \in \{1, \ldots, n\}$, σ_i is a dominant strategy.
- Best Response: A strategy σ_i of player i is a best response to their rivals' strategy profile σ_{-i} if $\forall \sigma_i' \in \Delta(S_i)$, we have $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})$
  - In finite strategic games of 2 players, a strategy is a strictly dominated strategy if and only if it is never a best response.[1].
- Rationalizable Strategies: A rationalizable strategy is a strategy that survives Iterative Elimination of Never Best Response Strategies.
- Nash Equilibrium: A strategy profile in which all strategies are best responses to each other. In games of finite **pure** strategies, an NE always exists, but it need not be PSNE.
  - As such, any strategy profile that does not survive IESDS cannot be in the set of Nash Equilibria.

[1] There's a more general version that covers actions for any number of players. See Osborne & Rubenstein p.60 Lemma 60.1 for detailed proof

2 Learning by Doing

1. (Rationalizability - MSU 2020 Midterm 1, Q2) Consider a three-player game: player 1 chooses one of the two rows, players 2 chooses one of the two columns, and player 3 chooses one of the four tables. All three players obtain the same payoffs given by numbers in the boxes below. Show that there is a strategy that is not strictly dominated but never a best response.

	L	R		L	R		L	R		L	R
U	8	0		4	0		0	0		3	3
D	0	0		0	4		0	8		3	3
	M_1			M_2			M_3			M_4	

2. (MSU Midterm 2023) The city of Rome has three alternatives:

   - X_1: Build a new metro line for sure.
   - X_2: Start building a new metro line but abandon the project if antique ruins are discovered (which occurs with probability $\pi \in (0, 1)$).
   - X_3: Stick to the status quo.

   There are three representative voters A, B, and C, who each represent $\frac{1}{3}$ of the population. Group i's valuation of alternative X_k is $v_i(X_k)$ for each $i \in \{A, B, C\}$ and $k \in \{1, 2, 3\}$. Assume that the payoff functions satisfy:

   - $v_A(X_1) > v_A(X_2) > v_A(X_3)$
   - $v_B(X_2) > v_B(X_3) > v_B(X_1)$
   - $v_C(X_3) > v_C(X_1) > v_C(X_2)$

   Consider a two-stage majority voting game: At stage 1, the representatives vote on whether or not to initiate the project. If the majority (2 out of 3) votes NO, then X_3 is chosen and the game ends. If the majority votes YES, then the game enters stage 2 where a vote is held to decide between X_1 and X_2.

   (a) Write down the normal form representation of this game.

(b) Suppose that the first stage of voting passed so that you only have to think about the second stage as the game. Find all the PSNEs in this second stage game (This is called a subgame).

3. (Simultaneous move games - MSU 2019 Midterm 1, Q2) Suppose that two firms are involved in a dispute over time to drive the other out of the market. Each firm $i \in \{1, 2\}$ values the total market at $v_i > 0$. The cost of the dispute per unit time is one unit of payoff. That is, if a firm fights for length of time t, it costs the firm t. Each firm has to decide at what time (if at all) to stop the dispute. For ease of notation, denote s_i as the time that firm $i \in \{1, 2\}$ chooses to stop, and treat time as a continuous variable. If firm i concedes at time t_i and firm j has not yet conceded, firm i gains nothing but incurs cost of fighting, and firm j gains the total market but has to incur the cost of fighting till firm i concedes. If both firms concede at the same time t, each gains one-half of its total market value ($v_i/2$) and also incurs the cost of fighting till t.

   (a) Define this strategic situation precisely as a normal form game

   (b) Find all strictly dominated strategies for each firm

   (c) Find all Pure Strategy Nash Equilibria of this game

4. (MSU Prelim, FS 2014 Part II, Q1) In a joint venture project, $n \geq 2$ partners are to determine the amount of capital y to be invested in the company. They use the following rule. Simultaneously, each partner i submits a real number $s_i \geq 0$. The amount of capital is determined to be

$$y = \min\{s_1, ..., s_n\}$$

and each partner i pays his share $\frac{y}{n}$ of the investment cost. The payoff of partner i is

$$u_i(y) = \sqrt{y} - \frac{y}{n}$$

(a) Write this formally as a normal-form game. That is, define the set of players, each player's strategy set, and her payoff function.

(b) At what value of y is $u_i(y)$ maximized?

(c) Now suppose $n = 2$. Show that $s_1^* = s_2^* = 1$ is a (weakly) dominant strategy equilibrium of this game.

5. (MSU Prelim FS 2022) Consider the following variation of a typical exchange economy with two players P_1 and P_2: the players have preferences over two goods x and y; the initial endowment of P_1 and P_2 are $(1, 0)$ and $(0, 1)$ [the first coordinate represents amount of x and the second one represents amount of y]. Player i's utility function is

$$\min \{x_i, y_i\}$$

where x_i and y_i are P_i's consumption of x and y respectively. The exchange works as follows. Each player simultaneously hands over a non-negative quantity of the good he possesses (up to his entire endowment) to the other player.

   (e) Write this as a game in normal form, and find all Pure Strategy Nash Equilibria of this game.

(e) Find all Pure Strategy Nash Equilibria where players do not play any (weakly) dominated strategy.

3 Go The Extra Mile

1. (MWG 7.D.1) In a game where player i has N information sets indexed $n = 1, \dots, N$ and M_n possible actions at information set n. How many pure strategies does player i have?

2. (EC301 SP23) Alice and Barbara are playing a one-stage guessing game. Each must choose a real number between 1 and 4 (inclusive). Alice's target is to match Barbara's number. Barbara's target is to name twice Alice's number. Each receives $10 minus a dollar penalty that is equal to the absolute difference between her guess and her target. Solve this game by iteratively deleting dominated strategies. What should Alice and Barbara choose?

3. (IEWDS) Show that in the following game, the order of iterated elimination of weakly dominated strategy matters.

	L	R
T	1,1	0,0
M	1,1	2,1
B	0,0	2,1