[SP26] ECN 812B Recitation 10

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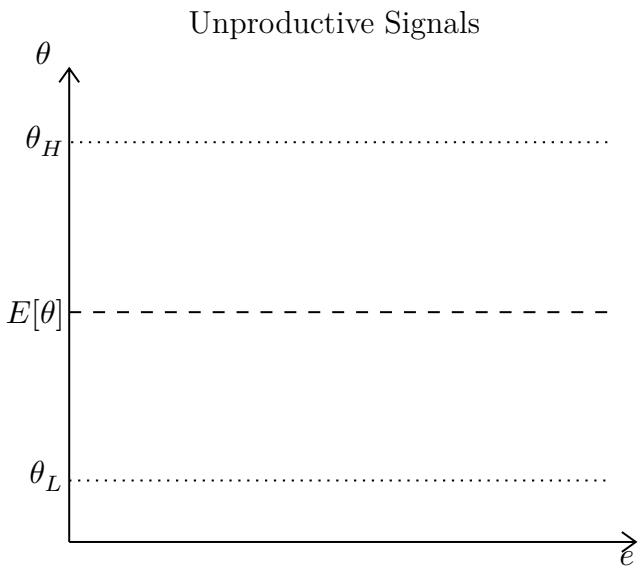
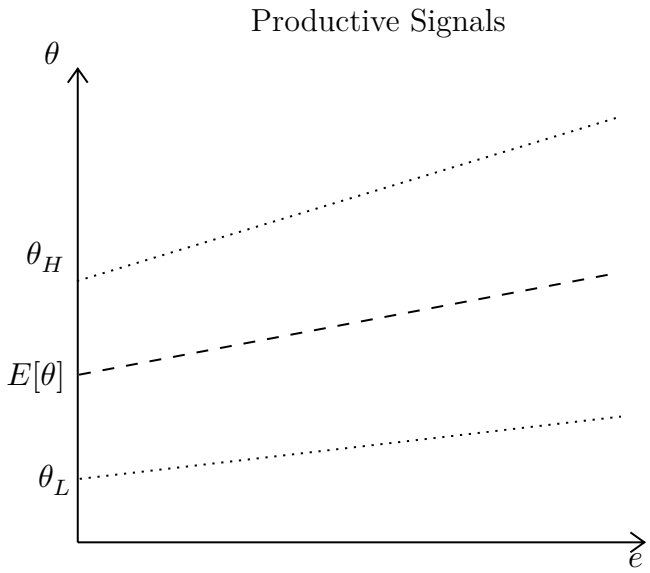
1 Concepts this Week

- Adverse Selection: The phenomenon where asymmetric information leads to inefficiency, and sometimes the complete "unravelling" of the market.
  - Instead of making decisions using the expected value of a good, buyers/sellers make decisions using the *conditional expected value* of a good.
  - For example, if the buyers only know a good is valued at $v \sim U[0,1]$ but seller knows thee exact value and will only sell if $p \geq v$. Then the expected value of the good priced at v is $E[v \mid v \leq p] = \frac{p}{2}$ instead of $E[v] = \frac{1}{2}$.
  - Sometimes market power is added to the questions, so one needs to be aware of the competitive environment.
  - Adverse selection need not be about buyers/sellers of goods. The model also applies to the labor market.
- Signaling: An application of wPBEs and PBEs where agents with private information can give a (costly) signal to reveal information.
  - Generally separated into (1) Separating Equilibrium and (2) Pooling Equilibrium
  - Incentive Compatibility: Each agent has the incentive to do what you want them to do. In a separating equilibrium, this means none of the types have an incentive to pretend to be another type.
  - Individual Rationality: Each agent has no incentive to **not** participate.


Unproductive Signals
θ
θ_H
E[θ]
θ_L
e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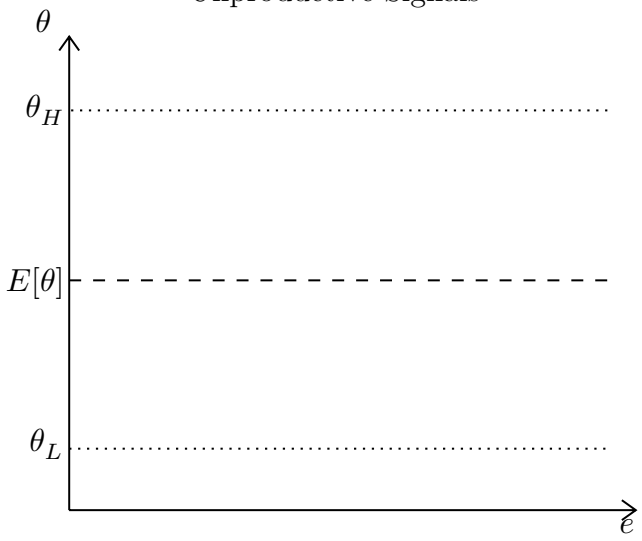
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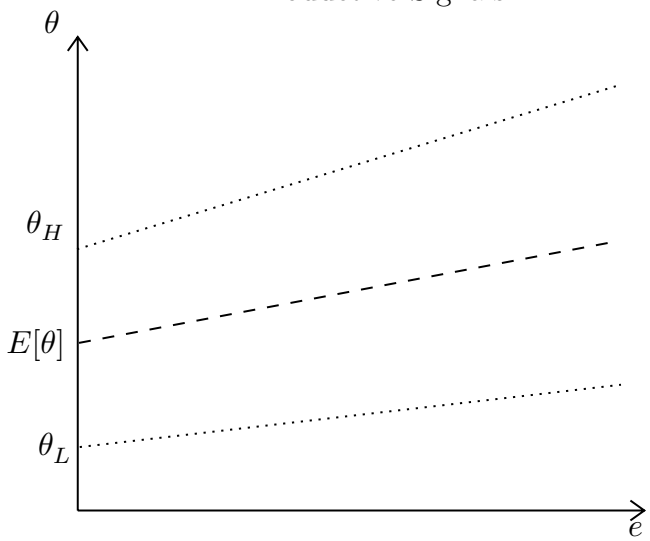
Productive Signals
θ
θ_H
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2 Learning by Doing

1. (MSU Prelim SS 2023 Q4, SS 2016 P2 Q3) Consider the Akerlof (1970) used car model. Each seller knows the quality of the car denoted by θ. Prospective buyers only know the distribution of θ, which is uniformly distributed on $[0, 1]$. If a prospective buyer purchases a car of quality θ at the price of p, his utility is given by $\theta - p$. If the buyer does not make a purchase, his utility is zero. Buyers are risk-neutral. There is a large number of prospective used car buyers (many more than potential sellers), which simply means that the market price is determined by the expected value of cars in the market to consumers. In contrast, a seller who has a used car of quality θ has the reservation utility of $r(\theta) = k\theta$ where $k \in [0, 1)$; he will only sell if the price is greater than or equal to $k\theta$.

   (a) Suppose that $k = \frac{1}{3}$. Is there an efficient market equilibrium? If so, describe it. If there is not, describe all inefficient equilibria.

(b) Suppose that $k = \frac{2}{3}$. Is there an efficient market equilibrium? If so, describe it. If there is not, describe all inefficient equilibria.

(c) For what values of k does there exist an efficient market equilibrium with the Pareto optimal number of trades?

(d) Assume that $k = \frac{2}{3}$. There is a test service that reveals the quality of a car perfectly. This service is *competitively* supplied at the marginal cost of $\frac{1}{4}$. Describe the market equilibrium.

(e) Continue to assume that $k = \frac{2}{3}$. Now suppose that there is a *monopolistic* supplier of the test (with the same marginal cost of $\frac{1}{4}$ as above). What price the monopolistic test provider would charge for such a service?

2. (MSU Prelim FS 2017 Q4) Consider the following variation of the Akerlof's lemon model. A seller sells an object of quality v to a buyer. The seller values an object of quality v at v but the buyer values it at θv where $\theta \in (1, 2)$. Moreover, the seller knows v whereas the buyer only knows that $v \sim U\left[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right]$ where ε is a parameter between $[0, \frac{1}{2}]$. The distribution of v is common knowledge. Notice that efficiency requires that all quality types are traded. Suppose that the seller has all bargaining power and sets a price that extracts all surplus from the buyers.

   (a) Find the equilibrium price.

   (b) How does the extent of inefficiency; i.e., the set of quality types that are not traded, vary with ε? Give a brief intuition for your finding.

3. (MSU Prelim SS 2012 P2 Q2) Suppose there are two types of workers differentiated by productivity level, θ. Good workers produce θ_H while bad workers produce θ_L where $\theta_H > \theta_L > 0$. Prospective employers cannot observe productivity. Suppose that $\Pr(\theta = \theta_H) = 1/2$. Suppose that prior to applying for jobs, workers can accumulate "ratings," $r \in \mathbb{R}_+$. The accumulation of ratings requires effort. In particular, suppose that a worker's preference is given by

$$u(w, r; \theta) = w - \mu r/\theta$$

where w is the wage payment and $\mu > 0$ is a parameter. The labor market is assumed to be competitive, that is, $w = \mathbb{E}(\theta \mid r)$. Answer the following questions. [*Hint:* It might help to approach this problems graphically as done in the class in the context of the Spence model.]

   (a) Identify the set of separating (weak)PBE. What is the most efficient separating equilibrium?

(b) Identify the set of pooling (weak)PBE. What is the most efficient pooling equilibrium?

(c) Consider the most efficient separating (weak)PBE. How do the equilibrium payoffs of the two types of the workers change when μ changes?

(d) Now, suppose that an unemployment benefit becomes available that gives a worker a payment of $B \in (\theta_L, \theta_H)$ if he does not accept employment. How will this benefit change the payoffs of the two types of the workers in the most efficient pooling and separating equilibria?

4. (MSU 2019 Final Q3) Consider a variation of the Spence signaling model where education is indeed productive. The worker can be one of two types: skilled (S) or unskilled (U). There is a prior probability $\frac{1}{2}$ that the worker is skilled. The worker can choose some education level e. The productivities are $2+e$ if the worker is skilled and $1+e$ if he is unskilled. The cost of education is $\frac{1}{4}e^2$ for a skilled type and $\frac{1}{2}e^2$ for an unskilled type. The worker learns his type, then chooses a level of education, the firm observes this choice and employs the worker at a wage w. The worker's payoff is wage minus the cost of education. The firm faces competition in the labor market and makes zero profit. That is, the worker's wage w is exactly his expected product (given his signal). [Note: This is a slightly generalized version of Spence model discussed in class. The only difference is that here the education is indeed useful in the production process—the worker's productivity increases in e.]

   (a) Suppose there is full information, i.e., the firm can observe the worker's type as well as education level. Find the educational level each type will choose in the SPNE.

(b) Assume asymmetric information: the firm cannot directly observe the worker's type. Characterize all the pooling weak PBE.

(c) Assume asymmetric information: the firm cannot directly observe the worker's type. Characterize all the separating weak PBE.

5. (BU Prelim 2018) Consider the following two player game. Player 1 is involved in an accident with player 2. Player 1 knows whether he is negligent or not. Player 2 does not know if player 1 is negligent, and assigns probability $\frac{3}{4}$ to him being negligent (these beliefs are common knowledge). If the case goes to court, the judge will learn the truth.

   Before the trial, player 1 sends a take-it-or-leave-it settlement offer to player 2, which can be either 3 or 5. Player 2 either accepts or rejects the offer. If he accepts it, the parties don't go to court and the game ends. If he rejects it, the parties go to court and player 1 has to pay 5 to player 2 if he is negligent and 0 if he is not negligent; in either case, player 1 has to pay court expenses of 6.

   The payoffs of the game can be epresented by the table:

P_1 \ P_2	Accept	Reject
3	$-3, 3$	$-6, 0$
5	$-5, 5$	$-6, 0$
	P_1 Not Negligent	

Accept	Reject
$-3, 3$	$-11, 5$
$-5, 5$	$-11, 5$
P_1 Negligent	

   (a) Draw the game tree of this game

(b) Find all the pooling pure strategy wPBE of this game.

(c) Construct a partially separating wPBE of this game in which P_1 plays a pure strategy when he is not negligent, and plays a mix strategy when he is negligent.

6. (MSU Prelim FS 2020 Q3) President Stanley announced a new opt-in option to NS/S grading policy during the coronavirus semester. Let's explore it and an alternative grading policy. Assume an individual's wage-earning ability is perfectly reflected by the grade, which will be a real number uniformly distributed between 0 and 4, $U[0, 4]$ (to clarify, any number between 0 and 4—3.45, 1.23, or 0.22—is possible). Suppose a student perfectly observes his grade θ, and the grade is posted in the university system.

   (a) He decides to disclose the exact grade, or pay a small cost $c \in (0, 1/2)$ to opt-in to disclose the grade as Satisfactory (S) if it is between 2 and 4 (inclusive), or to disclose the grade as Non-Satisfactory (NS) if it is between 0 and 2. Upon observing the grade, two firms compete for the student so that the student is paid the competitive wage based on the disclosed grade. Describe the PBE, i.e., student's equilibrium decision to disclose the grade, equilibrium wage, and firms' equilibrium belief.

(b) Suppose, instead of opting in the NS/S option, a student needs to pay a cost to opt *out* of the NS/S option: S is reported for any grade above 2 (inclusive) and NS is reported for any grade below 2 unless he pays a cost $c \in (0, 1/2)$ to report grade θ. Describe the PBE, i.e., equilibrium decision, wage, and belief.

3 Go the Extra Mile

1. (MSU 2020 Final Q2) A medical mask can be highly effective $\theta_H = 1$ or less effective $\theta_L = 0$ against the coronavirus, with an equal probability, $\lambda = \frac{1}{2}$. The general public buying the masks cannot distinguish the effectiveness of the masks. However, the manufacturers making effective mask for their better equipment and materials can more easily produce visibly more sophisticated mask wrappers (that by themselves are completely useless against the coronavirus). The cost of producing the mask wrappers of sophistication level e is given by $\frac{(2-\theta)e^2}{2}$. Two buyers are willing to pay up to θ for a type-θ mask, but they can only distinguish the effectiveness of masks but only the wrappers.

   (a) Describe a pooling PBE. (Hint: State equilibrium effort, price schedule, and belief both on and off equilibrium path. Graphs may help.)

   (b) Describe a separating PBE that does not survive the Intuitive Criterion. (Hint: State equilibrium effort, price schedule, and belief both on and off equilibrium path. Graphs may help.)

   (c) Describe a PBE that survives the Intuitive Criterion.

2. (Columbia Prelim 2011) Consider the following signaling game. Player 2 can be one of two types $t \in \{1, 2\}$, and they know t but P_1 only knows that $P(t = 1) > \frac{1}{2}$. The game under each type with associated payoff matrices is given below:

P_1 \ P_2	L	R
U	$-3, 3$	$-6, 0$
D	$-5, 5$	$-6, 0$
	$t = 1$	

L	R
$-3, 3$	$-11, 5$
$-5, 5$	$-11, 5$
$t = 2$	

   Suppose that P_2 can signal their type to P_1 by letting P_1 observe their action before $P - 1$ plays.

   (a) Show that there is a (weak) Perfect Bayesian Equilibrium in which the two types of P_2 pick the same signal. Is there more than one such equilibrium?

   (b) Show that there is a pure-strategy Perfect Bayesian Equilibrium in which the two types of P_2 pick different signals. Is there more than one such equilibrium?

   (c) Derive all mixed-strategy Perfect Bayesian Equilibrium in this game.