

[SP26] ECN 812B Recitation 11

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1 Concepts this Week

- **Strict Dominance** : Take some wage schedule set W^* . Effort level e is strictly dominated for type θ if $\exists e' \neq e$ such that

$$\inf \left\{ W^*(\Theta, e') - c(e', \theta) \right\} > \sup \left\{ W^*(\Theta, e) - c(e, \theta) \right\}$$

- Reasonable Type Set: The type set $\Theta^*(e)$ is a reasonable type set if it assigns strictly positive probability to type θ if and only if e is not strictly dominated for type θ .
- **Equilibrium Dominance**: A PBE effort level e is equilibrium dominated for type θ if:

$$u^*(\theta) > \sup \left\{ W^*(\Theta, e) - c(e, \theta) \right\}$$

where $u^*(\theta)$ is the equilibrium payoff of type θ

- A type set $\Theta^{**}(e)$ is **equilibrium reasonable** if

$$\Theta^{**}(e) \equiv \{\theta \mid e \text{ is not equilibrium dominated for } \theta\}$$

- Cho-Kreps: The PBE (σ, μ) violates the **Cho-Kreps Intuitive Criterion** if there exists e' and θ such that

$$u^*(\theta) < \inf \left\{ W^*(\Theta^{**}(e'), e') - c(e', \theta) \right\}$$

Cho-Kreps removes all inefficient separating PBEs as well as all pooling PBEs when the single-crossing condition holds.

In other words, a PBE violates the Cho-Kreps Intuitive Criterion if there is an off-path deviation that would yield a strictly better payoff for some type θ , assuming the receiver restricts their beliefs to only include types for whom that deviation is not equilibrium dominated.

- The key difference between equilibrium dominance and strict dominance is the benchmark used for comparison. Strict dominance requires an action to be unconditionally worse than some alternative action, regardless of receiver beliefs. Equilibrium dominance only requires an off-path action to yield a worse payoff than the status quo equilibrium payoff, $u^*(\theta)$, even under the most favorable receiver beliefs. Because comparing a deviation against a guaranteed equilibrium payoff is a weaker condition, it is much easier for an action to be equilibrium dominated than strictly dominated. Consequently, Cho-Kreps can eliminate types that strict dominance cannot.

2 Learning by Doing

1. Consider a game of charitable giving, in which donors care about the esteem that they receive from being considered generous by the public. There are two types of donors, generous (G) and miserly (M), which are equally likely in the population (i.e., the prior belief is $\alpha = P(G) = \frac{1}{2}$). The type of each donor is private information. Each donor chooses an amount x to donate to a charity. The individual donor's utility function is:

$$u_i(\theta_i, x, q) = -(x - \theta_i)^2 + q$$

where $\theta_i(G) = 1$ and $\theta_i(M) = \frac{1}{2}$. The term q is the expected esteem of the donor, where q is the public's equilibrium belief that the donor is generous. The timing of the game is as follows: Each donor chooses how much to donate to the charity $x \in [0, \infty]$, donations become publicly observable, and then the public forms a belief regarding each type of donor, and donors' payoffs are realized.

In what follows, let $x^*(\theta_i), q^*(x^*(\theta_i))$ denote the equilibrium donation by type θ_i and the corresponding equilibrium belief.

- (a) Find the range of donations sustainable in a pooling PBE defined by $x^P(G) = x^P(M) = x^P$ and $q^P(x) = 0$ for any $x \neq x^P$.

- (b) The Cho-Kreps intuitive criterion ensures that in equilibrium, $P(\theta_i | x) = 0$ for all x that are equilibrium dominated for type θ_i as long as x is not equilibrium dominated for all types. Recall that x^* is equilibrium dominated for type θ_i given $x^*(\theta_i), q^*(x^*(\theta_i))$ if $u(\theta_i, x^*, q^*) > u(\theta_i, x, q)$ for all q . Show that there is no pooling equilibrium that satisfies the Cho-Kreps intuitive criterion. (Note that the off-equilibrium beliefs may differ from the environment in the previous part.)

(c) Specify a fully separating PBE (x^G, x^M) that maximizes the individual expected donations and satisfies the Cho-Kreps intuitive criterion. Carefully specify the belief structure that supports this PBE.

(d) What is the individual expected donation under the equilibrium derived in the last part? What is the equilibrium payoff for each type?

Consider now the situation in which the charity employs category reporting instead of revealing the exact donation amount. The silver category includes donations $x < \hat{x}$ and the golden category includes donations that are at least as high as \hat{x} . The charity commits to disclosing only the category of the donor, not the actual donation made.

(e) Consider a fully separating PBE in which the generous type donates $x_C^S(G) \geq \hat{x}$ and the miserly type donates $x_C^S(M) < \hat{x}$. For what values of \hat{x} does such an equilibrium exist? How much does each donor donate in this equilibrium? Carefully specify the belief structure that supports this PBE and satisfies the intuitive criterion.

(f) What value \hat{x} generates the highest expected individual donation under a fully separating equilibrium such that the generous type donates $x_C^S(G) \geq \hat{x}$ and the miserly type donates $x_C^S(M) < \hat{x}$? Denote the value \hat{x}^* . Given \hat{x}^* and the corresponding equilibrium that you found, does category reporting generate higher or lower expected donations compared to the full disclosure separating equilibrium found in the original setting? Are the two types of donors better or worse off under category reporting?

2. Consider a used car market where there are three types of sellers: low, middle, and high.

A low seller values his own car at the reservation price \$2, but it is worth \$4 to a buyer.

A middle seller values his car at the reservation price \$10, but it is worth \$12 to a buyer.

A high seller values his car at the reservation price \$11, but it is worth \$14 to a buyer.

In the population of sellers, $\frac{1}{3}$ are low type, $\frac{1}{3}$ are middle type, and $\frac{1}{3}$ are high type. There are many more buyers than sellers in this market. Buyers cannot distinguish between different types of sellers. All individuals are risk-neutral.

(a) If buyers competitively announce price bids for cars, then what would be the highest possible equilibrium price?

- (b) Now consider instead a market in which each seller independently announces the price at which he will sell his car. If a buyer is willing to buy it at this price, then a transaction takes place. For this seller-offer game, find a partially separating equilibrium in which each type of seller offers a different price, and each type has a positive probability of trading. Among such separating equilibria, show the equilibrium in which the highest fraction of cars is expected to sell.

(c) For the seller-offer game that you considered in part (b), now consider equilibria in which all middle and high sellers offer the same price, but low sellers offer a different price. Among such equilibria in which the high and middle types pool together, show the equilibrium in which the highest fraction of cars is expected to sell.

(d) For each of the three equilibria that you found in parts (a), (b), and (c), identify whether it is (weakly) Pareto-dominated by one or more of the other two equilibria that you found. Justify your answer by appropriately comparing (interim) expected payoffs.

3. (Modified from Stanford Prelim 2003 Q2) A prospective economics PhD student can be of two types, L and H . A PhD program has to give competitive offers to prospective students and pay a type i student π_i such that $\pi_H > \pi_L$. The students know their own types but the program does not. The students can choose to do a pre-Doc program for length p , which costs the student $c_i(p)$. Notice that a pre-Doc experience does not increase the stipend of the student. Assume that $c_H(p) = p^2$ and $c_L(p) = p$. Throughout this question, focus only on pure-strategy equilibria.

(a) Does the cost functions given satisfy the single crossing property?

(b) Does a separating equilibrium exist in this model?

Suppose now that in addition to the student's education, the programs can also observe how much money d the student spent on partying during their senior year of college, and the student's utility function is $\pi_i - c_i(p) - d$.

- (c) Construct a separating equilibrium where the L type student does no pre-Doc and did not spend money on partying. What is the most efficient equilibrium?

- (d) What happens to the payoff of type H as π_H increases? Give an intuition of your finding.

4. (Modified UPenn Prelim 2016 Q3) Suppose that the payoff to a firm from hiring a worker of type θ with education e at wage w is

$$f(e, \theta) - w = 3e\theta - w.$$

The utility of a worker of type θ with education e receiving a wage w is

$$w - c(e, \theta) = w - \frac{e^3}{\theta}.$$

The worker's ability is privately known by the worker. There are at least two firms. The worker (knowing their ability) first chooses an education level $e \in \mathbb{R}_+$; firms then compete for the worker by simultaneously announcing wage; finally, the worker chooses a firm. Treat the wage determination as in class, a function $w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ determining wage as a function of education.

Suppose the support of the firms' prior beliefs ρ on $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L = 1$ and $\theta_H = 3$.

- (a) Suppose that the firms can perfectly observe the worker's type. What is the most efficient level of education for each type of worker?

- (b) Is there a perfect Bayesian equilibrium in which both types of worker choose their full information education level? Be sure to verify that all the incentive constraints are satisfied.

- (c) What is the lowest possible $\rho = P(\theta = \theta_H)$ for there to exist a pooling equilibrium where both types of workers choose the same education.

3 Go the Extra Mile

- (Modified from NYU Prelim 2014 Q4) There is one seller with a car for sale, with quality $q \sim U[0, 1]$. There is one potential buyer. If a car of quality q is sold at price p , then the seller's payoff is p and the buyer's payoff is $v(q) - p$ where $v(\cdot)$ is a known non-negative strictly increasing function. If the seller keeps the car, then the seller's payoff is q , and the buyer's payoff is 0.
 - Suppose the seller announces a price and the buyer either accepts or rejects, what is the equilibrium price?
 - Suppose that the seller can purchase a signal $s \in \mathbb{R}^+$ before announcing a sale price: signal s costs $\frac{s}{q}$ for a seller with a car of quality q . After seeing both the price and the signal, the buyer can either accept or reject the price. Assume that $v(q) = q + b$, $b > 0$. Does there exist a separating PBE where all cars with quality q are sold at $v(q)$. Does your answer depend on b ?
- (Penn Prelim FS 2017 Q3) An entrepreneur is contemplating selling all or part of his startup to outside investors. The profits from the startup are risky and the entrepreneur is risk averse. The entrepreneur's preferences over $x \in [0, 1]$, the fraction of the startup the entrepreneur retains, and p , the price "per share" paid by the outside investors, are given by

$$u(x, \theta, p) = \theta x - x^2 + p(1 - x)$$

where $\theta > 1$ is the value of the startup (i.e., expected profits). The quadratic term reflects the entrepreneur's risk aversion. The outside investors are risk neutral, and so the payoff to an outside investor of paying p per share for $1 - x$ of the startup is then

$$\theta(1 - x) - p(1 - x)$$

There are at least two outside investors, and the price is determined by a first price sealed bid auction: The entrepreneur first chooses the fraction of the firm to sell, $1 - x$; the outside investors then bid, with the $1 - x$ fraction going to the highest bidder (ties are broken with a coin flip). Important convention: The outside investors submit bids in "price per share" p , so the amount paid is $p(1 - x)$.

- Suppose θ is public information. What fraction of the startup will the en-

trepreneur sell, and how much will he receive for it?

- (b) Suppose now θ is privately known to the entrepreneur. The outside investors have common beliefs, assigning probability $\alpha \in (0, 1)$ to $\theta = \theta_1 > 1$ and probability $1 - \alpha$ to $\theta = \theta_2 > \theta_1$. Suppose $\theta_2 - \theta_1 > 2$. Characterize the separating perfect Bayesian equilibria. Are there any other perfect Bayesian equilibria?