[SP26] ECN 812B Recitation 12

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1 Concepts this Week

- Individual Rationality: Every participant's utility is at least as good as their outside option $\bar{u}$ (typically normalized to 0)
- Incentive Compatibility: Every participant's utility of truthfully revealing their type is at least as good as misreporting their type
- Screening
  - In signaling, the agents pay the cost of signal to separate themselves. In screening, the principal pays the agents to self-separate.
  - First-Best: The solutions when there is no information asymmetry.
  - Second-Best: The most efficient contract under information asymmetry.
- Differences between concepts:
  - Adverse Selection:
  - Signaling:
  - Monopolistic Screening:
  - Moral Hazard (next week):

2 Learning by Doing

1. (MSU FS2020 Q3) A third-year college athlete decides whether to forgo her final year of the four-year college eligibility to become a professional athlete. Her payoff is the sum of her athletic earnings and her after-sports earnings. She knows her own true athletic ability $\theta \in [0, 1]$, but her future athletic employer will not know her true ability if she turns pro early. If she becomes a professional athlete early, her athletic earning is (endogenously) determined by the average ability of all those who turn pro early. If she waits one more year, her true ability is fully revealed and her athletic earning equals her athletic ability. The athletic abilities of all college athletes are public known to be uniformly distributed between 0 and 1. Besides the athletic earnings, she will be paid based on her education level after she retires from sports: She will receive wage earnings w_L if she turns pro early, and wage earnings $w_H > w_L$ if she stays fourth year and receives a college degree. If she waits one more year, her athletic earnings and wage earnings are discounted by $\delta \in (\frac{1}{2}, 1]$. In summary, her payoff is $\delta(v + w_H)$ if she stays one more year, and is $v + w_L$ if she turns pro early, where v is her equilibrium athletic earnings.

   (a) Describe the competitive equilibrium under different parameters, specifying her equilibrium decision and equilibrium wage if she turns pro early.

(b) Suppose an athlete can pay an extra cost $c > 0$ (e.g., hire an agent, showcase her ability in pre-draft trials) to reveal her true ability. Assume $w_L > \delta w_H$. Describe a competitive equilibrium in which all three decisions (i.e., turning pro early without paying, turning pro with a cost, or waiting) are present in equilibrium, specifying her equilibrium decision and equilibrium wage if she turns pro early without paying the extra cost. Characterize the condition that such an equilibrium exists.

2. (MSU 2021 Final Q3) An agent privately observes her income 0 (low) or 1 (high). It is common knowledge the income is either 0 or 1 with equal probability. The principal (IRS) decides the income tax rate $t \in [0,1]$. A high-income agent can misreport her income to be low; while the low-income agent can only tell the truth. To prevent the high-income agent's misreporting, the principal audits the agent's income with probability $p \in [0,1]$ when it is reported to be low. Assume that auditing costs $c \in (0,1)$. When the agent is caught for misreporting, her income will be taken away. Suppose the agent is risk averse and has the utility function $u(x) = \sqrt{x}$ and the principal is risk neutral.

   (a) Suppose that the principal can commit to a policy (t, p) to maximize its expected payoff. Formulate and solve the principal's problem.

   (b) Is the ex ante optimal p you solved in (a) sequentially optimal? Why or why not, explain your reasoning.

(c) From now on, assume that $t = 0.5$, $c = 0.2$ and suppose that the principal cannot commit to any auditing strategy ex ante. He will make such a choice after receiving the agent's income report. Solve for the mixed-strategy perfect Bayesian equilibrium where the high type agent reports each type of income with strictly positive probability.

3. (MSU 2019 Final Q4) Suppose that a monopolist seller wants to sell a unit of a good to a buyer. The buyer's valuation of good is her private information but it is commonly known that the buyer can have one of two valuations: θ_l (with probability p_l) or θ_H (with probability p_h), where $0 \leq \theta_l < \theta_h$. While the seller is risk neutral, assume that the buyer is risk averse. Moreover, suppose that the seller uses a lottery to sell the good where the lottery is defined as follows: Trade takes place if the buyer wins the lottery. The lottery specifies a winning probability q for the buyer, a price of the good W if the buyer wins, and a penalty L if the buyer loses. A bidder has utility $u(\theta - W)$ when winning and paying W, and $u(-L)$ when losing and paying L. The utility function is strictly increasing and concave. Let the reservation utility of the buyer be $u(0)$.

Now assume that the seller offers a menu of lotteries $\{(q_l, W_l, L_l), (q_h, W_h, L_h)\}$ to the buyer where (q_i, W_i, L_i) is the lottery intended for the buyer of type θ_i, $i \in \{l, h\}$. Assume the production cost is zero and the seller's reservation price is zero. Furthermore, assume $p_l q_l + p_h q_h \leq \frac{1}{2}$ and $q_h \leq p_l + \frac{p_h}{2}$. The optimal screening problem for the seller can now be written as:

$$\max_{\{(q_l, W_l, L_l), (q_h, W_h, L_h)\}} p_l \left[q_l W_l + (1 - q_l) L_l\right] + p_h \left[q_h W_h + (1 - q_h) L_h\right]$$

subject to

$$q_l u(\theta_l - W_l) + (1 - q_l) u(-L_l) \geq u(0) \qquad (IR_L)$$

$$q_h u(\theta_h - W_h) + (1 - q_h) u(-L_h) \geq u(0) \qquad (IR_H)$$

$$q_l u(\theta_l - W_l) + (1 - q_l) u(-L_l) \geq q_h u(\theta_l - W_h) + (1 - q_h) u(-L_h) \qquad (IC_L)$$

$$q_h u(\theta_h - W_h) + (1 - q_h) u(-L_h) \geq q_l u(\theta_h - W_l) + (1 - q_l) u(-L_l) \qquad (IC_H)$$

The following questions asks you to characterize the optimal screening contract.

(a) Argue that that in the reduced program, the low type's IR constraint and the high type's IC constraint must bind.

(b) Show that in the reduced program the high type is fully insured (i.e., we must have $-L_h = \theta_h - W_h$)

(c) Show that in the reduced program the high type's IR constraint is slack (strict inequality holds) if and only if $L_h < 0$

(d) Show that in the reduced program we must have $L_l > 0$

4. (Modified MSU FS 2018 Q5) A principal needs to buy the services of a skilled artisan and needs to design an optimal contract when the artisan's cost of supplying his services is not known by the principal. If the artisan's cost type is t, then his net utility gain from providing q hours of service from either type would be $u(w, q|t) = w - tq$. The principal's net benefit from paying w for q hours of service from the wage can depend on the artisan's cost type (which the artisan knows), but the artisan could misrepresent his type. The artisan will not work until he gets a non-negative utility gain.

   (a) Suppose that the artisan's cost type is either 3 or 5, and the principal thinks that the probabilities are $\Pr(t = 3) = 2/3$ and $\Pr(t = 5) = 1/3$. Find the contract that maximizes the principal's expected net benefit from her business with the artisan, where the principal's gain from trade is $6\sqrt{q} - w$. What is the optimal menu of contracts $\{(w(t), q(t))\}$?

Suppose now it is common knowledge that the artisan's cost type t is drawn from $U[3,5]$, so that the principal's objective is to choose $(w(t), q(t))$ such that it maximizes

$$\int_3^5 \frac{6\sqrt{q(t)} - w(t)}{2}\, dt$$

(b) Argue that if the menu $\{(w(t), q(t))\}$ is incentive compatible, then

$$t \in \underset{\tau \in [3,5]}{argmax}\; w(\tau) - tq(\tau)$$

and use this fact to show that the optimal contract is such that

$$w(t) = u(w(5), q(5) \mid t = 5) + tq(t) + \int_t^5 q(s) ds$$

(c) Using the formula for $w(t)$, argue whether $t = 5$ artisan should get any information rent.

(d) Given the above formula for w(t), the principal's payoff can be written as:

$$\int_{3}^{5} \frac{6\sqrt{q(t)} - (2t-3)q(t)}{2} \, dt$$

Derive the formula for the optimal $q(t)$. What is $q(3)$ under hits formula? What can you say about distortion for the $t = 3$ type artisan under this optimal contract?