[SP26] ECN 812B Recitation 13

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1 Concepts this Week

Moral Hazard/**Hidden Action**/Principal-Agent Problem

- In screening (**hidden types)**, the principal pays the cost for agents to self-separate. In moral hazard, the principal doesn't need to pay the cost for agents to self-separate because there is no types. However, there are different levels of efforts that the agent can exert, and the principal's goal is to incentivize the agents to exert the effort that the principal wants.
- The principal *contract* on observables, whether that is outcome, signal, or something else.
  - An object is said to be *contractible* if the payoff can be written as a function of that object.
  - For example, in a course, we contract your final grade (analogous to wage) on your over exam grade (observable signal of your effort).
- Individual Rationality: Every participant's utility is at least as good as their outside option $\bar{u}$ (typically normalized to 0).
- Incentive Compatibility: Every participant's **expected utility** of exerting high effort is at least as good as that of exerting low effort.
- If the agent is risk-neutral, then first-best is achievable
- If the agent is risk-averse, then first-best is not achievable as the firm needs to take on more risk to incentivize/implement a certain effort level.

Final exam is 7:45-9:45 am on May 1st in regular classroom. Good luck on your finals/prelims!

2 Learning by Doing

1. (MSU Final 2020) Consider the following moral hazard problem. There is a firm and a single worker who can either exert effort (e = 1) or shirk (e = 0). If the worker exerts effort, the output is H = 10. If the worker shirks then the output, π, is 10 with probability p and $L \geq 0$ with probability $1 - p$ such that $p \cdot 10 + (1 - p) \cdot L = 5$. The distribution of output conditional on the worker's effort choice is common knowledge. First, the owner of the firm offers a wage schedule $(w(H), w(L))$ such that $w(\pi) \geq 0$ for both $\pi = H, L$. Then the worker decides whether to exert effort or shirk. If the realized output is π, the owner's payoff is $\pi - w(\pi)$ and the worker's payoff is $w(\pi) - 3e$. Both the owner and the worker are expected payoff maximizers.

   (a) Show that if the owner wants to implement $e = 1$, the worker's incentive constraint binds.

(b) Show that if the owner wants to implement $e = 1$, then $w(L) = 0$. Derive $w(H)$ in this case.

(c) For what values of (p, L) does the owner want to implement $e = 1$?

(d) Suppose that prior to interacting with the owner, the worker can choose any pair (p, L) as long as $p \cdot 10 + (1 - p) \cdot L = 5$. This choice is observed by the owner before he determines the wage schedule. What is the equilibrium choice of the worker?

2. (MSU Prelim FS 2015 P2) A principal P hires an agent A to work on a consulting project. The outcome of the project is $Y \in \{0, y\}$; think of y (> 0) as "success" and 0 as "failure". Y is publicly observed and contractible. A is required to exert costly effort $e \in \{0, 1\}$ to first learn the "state of the world" $\theta \in [0, 1]$ and then perform a task $t \in [0, 1]$. But A can always choose a t even without knowing θ. Cost of effort $e = 1$ is c $(< y)$ and the cost of $e = 0$ is 0. Also, all tasks can be performed at zero cost.

   The project is surely successful, i.e., $Y = y$ if $t = \theta$. If A performs any task $t \neq \theta$, $Y = 0$ except for the following case: there exists a task $t^* \in [0, 1]$ such that if $t = t^*$, for any θ, $Y = y$ with probability p and $Y = 0$ with probability $1 - p$. The agent, however, does not know which task is t^* but P has this information. Note that P does not observe e, the task chosen by A or θ.

   To understand this setting, consider the following scenario: think of θ as the source of the problem that the client is facing. A must exert effort to learn the nature of the problem and then suggest the appropriate strategy to address the concern. Once the problem source is identified, it can surely be fixed by choosing the appropriate strategy. P observes if the client's problem has been solved but does not know what the source was or what strategy A has recommended to address the issue. However, P knows that one of the available strategies is "special" in the sense that irrespective of the source of the client's problem, if this strategy is prescribed, there is some chance that it would resolve the problem. But A does not know which one of the all available strategies is the "special"one.

   P offers a salary $s \geq 0$ and bonus reward $b \geq 0$ if $Y = y$. That is, A's wage contract is of the form: $w(0) = s$ and $w(y) = s + b$. P's payoff is $Y - w(Y)$ and A's payoff is $w(Y) - ce$. The outside options for both P and A is 0.

(a) Suppose P wants to implement $e = 1$ by choosing s and b. Write down P's optimal contracting problem. Clearly specify P's objective function and **explain** the associated (IC) and (IR) constraints (recall that $s, b \geq 0$ must also be included in the set of constraints). (You must explain your derivation of the (IC) to be consider for any credit in this question).

(b) Solve for the optimal contract and compute P's equilibrium payoff. [*Hint:* At the optimum, $s = 0$.]

(c) Now suppose P reveals to A which task is t^*. Write down P's optimal contracting problem that implements $e = 1$. Clearly **explain** the (IC) and (IR) constraints and solve for the optimal contract. What is P's equilibrium payoff? (You must explain your derivation of the (IC) to be consider for any credit in this question).

(d) It is often argued that more transparency in organization is better for the business. Using your findings in (ii) and (iii), show that, contrary to this argument, letting A know about t^* hurts P and if p is large, no effort could be induced in equilibrium. Also, provide a brief intuition behind your finding.

3. (MSU Prelim SS 2017 Q4) Consider a three-action, two-outcome principal-agent problem. The outcomes are denoted by $\pi_H = 10$ and $\pi_L = 0$, which represent gross profits to the principal. The three effort levels are denoted by as $a_H = \frac{20}{12}$, $a_M = \frac{19}{12}$ and $a_L = \frac{16}{12}$. The agent's utility function is of the form $U(w, a) = \sqrt{w} - a$, where a denotes effort, and w is the wage payment. Suppose that the the agent's reservation utility is 0. The effort level a_i results in outcome π_j with probability $f\left(\pi_j \mid a_i\right)$, where $f(\pi_H \mid a_H) = \frac{2}{3}$, $f(\pi_H \mid a_M) = \frac{1}{2}$, and $f(\pi_H \mid a_L) = \frac{1}{3}$.

   (a) What is the optimal contract when effort is observable?

   (b) What is the optimal contract when effort is not observable? [Hint: Not all effort levels may be implementable.

4. (MSU SS 2013 Part II Q4) Consider a two person partnership problem. Output is generated by the simultaneously chosen efforts of two agents. Specifically, if agent 1 chooses $e_1 \geq 0$ and agent 2 chooses $e_2 \geq 0$, their joint output is a deterministic function $F(e_1, e_2)$ of their efforts. Each agent i bears a cost of her own effort of $c(e_i) = \frac{1}{2}e_i^2$. The output is publicly observable.

   (a) Suppose $F(e_1, e_2) = e_1 + e_2$ and that the effort is observable. Find the first-best effort choices (e_1^*, e_2^*), i.e., those that maximize the output net of the agents' costs.

(b) Suppose $F(e_1, e_2) = e_1 + e_2$ as before, but the effort is not observable. Suppose further that output must be shared in a budget balancing way, i.e., if the output turns out to be x and agent 1 receives $s(x)$, then agent 2 receives $x - s(x)$. The agents choose effort levels that form a Nash equilibrium of the game with payoffs

$$u_1\left(e_1, e_2\right) = s(F(e_1, e_2)) - c(e_1)$$
$$\text{and } u_2\left(e_1, e_2\right) = F(e_1, e_2) - s(F(e_1, e_2)) - c(e_2)$$

Assume that $s(\cdot)$ is differentiable. Show that the first best cannot be an outcome of any pure-strategy Nash equilibrium.

These F.O.C.s yield:

$$e_1^* = s'(F^*) \text{ and } e_2^* = 1 - s'(F^*)$$

The first-best outcome is $e_1^{FB} = e_2^{FB} = 1$, which will violate this condition. As such, first-best outcome is not achievable in this case.

(c) Now suppose that $F(e_1, e_2) = \min\{e_1, e_2\}$, and the cost functions are unchanged. Find the first-best allocation, and show that it can be achieved in a Nash equilibrium with a sharing rule according to which each agent receives half of the output, i.e., $s(x) = \frac{1}{2}x$.