[SP26] ECN 812B Recitation 2

Willy Chen

January 30, 2026

1 Concepts this Week

- Nash Equilibrium: A strategy profile in which all strategies are best responses to each other. In games of finite pure strategies, an NE always exists, but it need not be PSNE.
  - As such, any strategy profile, that does not survive IESDS, cannot be in the set of Nash Equilibria.
- Monopoly ($|\mathcal{J}| = 1$)
- Bertrand Model of **Price** Competition ($|\mathcal{J}| \in \mathbb{N}$)
  - Bertrand equilibrium in symmetric games results in 0 profit for all sellers.
- Cournot Model of **Quantity** Competition ($|\mathcal{J}| \in \mathbb{N}$)
- The key to solving a lot of game theory problems is to properly differentiate, in notation, a specific strategy from a general strategy ($s_i = s$ vs. s_i).
- A **Mixed Strategy Nash Equilibrium** always exists in finite strategic games
  - An MSNE to player i is a mixed strategy that makes player $-i$'s indifferent between their own pure strategies by creating "certainty" from uncertainty.

2 Learning by Doing

1. (Modified MSU SS23 Midterm Q3) Consider a symmetric Bertrand duopoly problem with heterogeneous commodities. The commodities are imperfect substitutes. Suppose that firm $i, j \in \{1, 2\}$ with $i \neq j$ has the following demand functions:

$$D_i(p_i, p_j) = 1 - p_i + \frac{1}{2}p_j$$

where p_i and p_j are prices of firms i and J, respectively. Marginal cost of production is constant $c = \frac{1}{4}$.

   (a) Write down firm i's profit function.

   (b) Suppose the two firms choose prices simultaneously. What are the Nash equilibrium prices and firm profits?

(c) Suppose now that firm 1 sets p_1 first and after observing p_1, firm 2 chooses p_2. What are the Nash equilibrium prices and firm profits?

2. (MSU Prelim, SS 2016, Pt II Q2) Consider the following modification to Cournot competition game: There are two stages. In stage one, $|\mathcal{J}| = N$ firms simultaneously decide whether to enter the market or not. If a firm enters, it incurs an entry cost of F. In stage two, the entrants compete in the Cournot fashion.

   Suppose that all firms have identical cost function $c(q)$ and the market inverse demand function is given as $p(q)$. If there are n firms in the market, denote the Nash Equilibrium quantity of firm j in the second stage as $q_j(n)$. Hence, the total quantity is $\sum_{i \in \mathcal{J}} q_j i(n)$ and stage two profit for each firm is:

$$\pi_j(n) = p\left(\sum_{i \in \mathcal{J}} q_i(n)\right) q_j(n) - c(q_j(n))$$

   Assume that $\pi_j(n)$ and $q_j(n)$ are both decreasing in n and a firm's outside option is 0. Answer the following questions about the firms' entry decision in stage one.

   (a) Suppose that there exists a pure strategy equilibrium where exactly n^* firms enter in stage one. Write down the condition on firm j's profit $\pi_j(n)$ that must be satisfied at n^*.

   (b) Now suppose that there exists a mixed strategy equilibrium where each firm enters with probability α^*. Write down the condition on firm's profit that must be satisfied at α^*.

(c) Suppose that the social planner (SP) can choose the number of firms in stage one and then firms compete a la Cournot in stage 2. SP wants to choose n to maximize aggregate surplus (consumer and producer surplus net of cost of entry) given the competition in stage two. Write down SP's problem and derive the first-order condition.

(d) Using your answers in (a) and (c), show that there can be excess entry in equilibrium in comparison to the socially optimal level.

3. (WN20 UChicago ECON 302 PS6. This question is motivated by empirical work of Pierre-Andre Chiappori.) Consider the penalty kick in soccer. There are two players, the goalie (G) and the kicker (K). The kicker has two strategies: kick to the goalie's right (R) or to the goalie's left (L). The goalie has two strategies: move left (l) or move right (r). Let α be the probability that the kick is stopped when both choose left and let β be the probability that the kick is stopped when both choose right. Assume that $0 < \alpha < \beta < 1$. (Consequently, the kicker is more skilled at kicking to the goalie's left.) The payoff matrix is as follows.

G \ K	L	R
l	$\alpha, 1-\alpha$	$0, 1$
r	$0, 1$	$\beta, 1-\beta$

(a) *Before analyzing this game*, informally answer the following questions.

(i) Would you expect a kicker who is more skilled at kicking to the goalie's left than to his right, score more often when he kicks to the goalie's left?

(ii) If a kicker's ability to score when kicking to the goalie's left rises (i.e. α decreases), will it affect the percentage of times the kicker scores when he chooses to kick to the goalie's left? Will it affect his scoring percentage when he kicks right?

(b) Find the unique Nash equilibrium of this game

(c) Answer again the questions in part (a). Based upon this, would it be wise to judge a kicker's relative scoring ability in kicking left versus right by comparing the fraction of times he scores when he kicks right versus the fraction of times he scores when he kicks left?

4. (MSU 2020 Midterm 1, Q3) Suppose that N players are bidding in an *all-pay* auction for an object that every bidder values at $V > 0$. The rules of the auction are as follows:

   - Each bidder submits her bid in a sealed envelope to the auctioneer (so a bidder does not know others' bids when he is placing his bid)
   - Auctioneer opens all bids at the same time, and gives the object to the highest bidder.
   - Each bidder—including all the losing ones—pays her bid amount to the auctioneer.

   In case of a tie, each of the highest bidders has an equal probability of getting the object.

   (a) Show that there does not exist any pure strategy NE of this game.

   (b) Suppose every player $j \neq i$ chooses her bid b_j according to the distribution $F(x) = Pr(b \leq x), x \in [0, V]$, where the associated pdf f has full support $[0, V]$. Write out player i's expected payoff from choosing a bid x.

(c) Find a symmetric mixed strategy NE of the game. That is, suppose in NE, every player chooses her bid b according to the distribution $F(x) = Pr(b \leq x), x \in [0, V]$, where the associated pdf f has full support $[0, V]$. Find $F(x)$.

3 Go the Extra Mile

1. (Simultaneous move games - MSU 2012 Midterm 1, Q1) Suppose there are n witnesses to a crime and they must decide simultaneously whether to report it to the police or ignore the incident. Calling the police has a cost of 1 (you can either interpret it as the disutility of effort needed to report the crime to the authority and/or the disutility of the possible future involvement with the police investigation). All witnesses prefer to call the police than to let the crime go unreported. If at least one witness calls the police, all witnesses get a benefit of $x > 1$. So, the payoff of the caller is $x - 1$ whereas the payoff of everyone else is x. If no one calls the police, everyone gets a payoff of 0.

   (a) Find all pure strategy Nash equilibria of this game.

   (b) Derive a symmetric mixed strategy Nash equilibrium of this game where each witness calls the cops with probability p.

   (c) Show that p is decreasing in n and increasing in x. What is the economic intuition behind this finding?