[SP26] ECN 812B Recitation 6 (Midterm Review)

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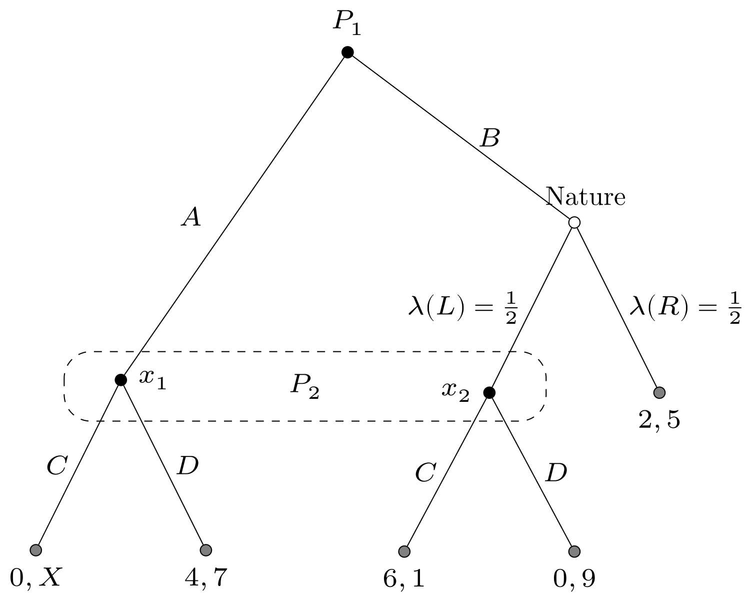
1. (Chicago Prelim 2016) Consider the following extensive-form game, which begins with a move by player 1. [*Hint: Not all parts require calculation. You can cite known relations between different equilibrium concepts.*]



   (a) Suppose that $X = 9$ is common knowledge. Construct a normal form representation of this game. A payoff table suffices.

(b) Find all Nash equilibria of this game.

(c) Show that any strategy profile that involves either player playing a pure strategy cannot be part of a *Weak Perfect Bayesian Equilibrium*.

(d) Find a *Weak Perfect Bayesian Equilibrium* for this game.

(e) Is the Weak Perfect Bayesian Equilibrium you found in part (d) also a *Perfect Bayesian Equilibrium*? Why or Why not?

(f) Is the Weak Perfect Bayesian Equilibrium you found in part (d) a *Sequential Equilibrium*? Prove by either constructing a sequence of completely mixed strategies or show that there does not exist such sequence. h nj b

2. (BU Prelim SS 2019) The fraction $\alpha \in (0, 1)$ of a population (of size N) of agents are the low (L) type, the remaining are of high (H) type. Each agent is privately informed about her type. There are two dates $t = 0, 1$. A pair of agents can form to create a group at date t. The payoff of an i type agent in forming a group with a j type agent at date t is $\delta^t B_{ij} > 0$ (where $\delta \in (0, 1)$). Her payoff is 0 if she does not form a group with anyone at either date. Everyone prefers to form a group with an H type, and H types have a higher marginal utility of pairing with another H type:

$$B_{HH} - B_{HL} > B_{LH} - B_{LL} > 0$$

The group formation process is as follows. At date $t = 0$, all agents are randomly matched with one another; an agent cannot verify the type of the other agent she is matched with. Each matched agent decides whether to accept or reject the other agent she is currently matched with. If both matched agents accept each other, they form a group at $t = 0$. Agents that do not form groups at $t = 0$ wait until $t = 1$, when all remaining agents are randomly matched with one another to form groups. The strategy of each agent is whether to accept or reject the agent she is matched with at $t = 0$.

(a) Show that there is an equilibrium where every agent accepts the partner they are matched with at date 0.

(b) Show that if

$$\frac{B_{HH}}{B_{HL}} > \frac{1-\delta\frac{\alpha}{\alpha+1}}{\delta\left((1-\frac{\alpha}{\alpha+1}\right)} > \frac{B_{LH}}{B_{LL}}$$

and if N is large enough, then there exists another equilibrium where L types accept, while H types reject the agent they are matched with at date $t = 0$.

3. This problem is a variation on cheap–talk games. P_1 has type $t \in \{0, 100\}$ where the common prior is that both types are have probability $\frac{1}{2}$. P_2's set of actions is $\mathbb{R}_+$. The rules of the game is as follows:

   Stage 1: Before player 1 learns their type, they choose whether to invest or not. If P_1 invests, this costs them some constant $c \in (0, 25)$. If they do not, then $c = 0$. The investment decision is only known to P_1.

   Stage 2: After investment decision, P_1 learns their set of feasible messages. If they did not invest, their set is $\{m_\emptyset\}$. If they invested, then their set is $\{m_t, m_\emptyset\}$ where t is their type. From the set of feasible messages, P_1 sends a message to P_2.

   Stage 3: P_2 sees ONLY the message sent by P_1 and nothing else. P_2 chooses an action $a \in \mathbb{R}_+$ and the game ends. P_1's payoff is $a - c$ and P_2's payoff is $-(t - a)^2$.

   (a) Is there a pure strategy PBE where P_1 does not invest? If so, give such an equilibrium. If not, explain why not.

(b) Is there a pure strategy PBE where P_1 invests? If so, give such an equilibrium. If no, explain why not.