[SP26] ECN 812B Recitation 4

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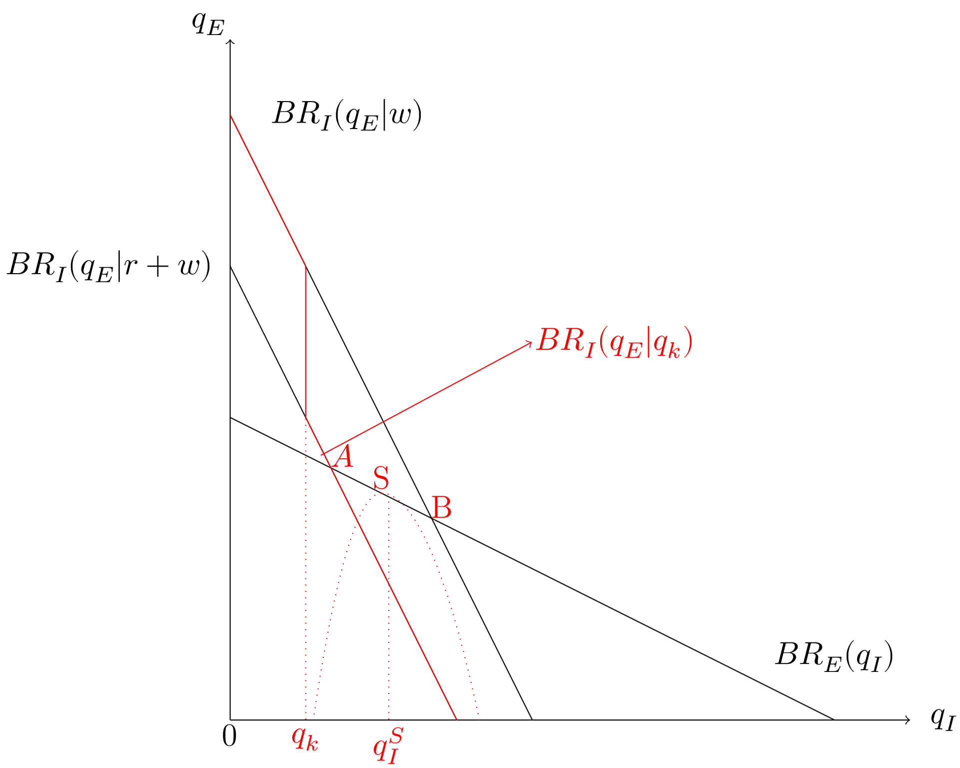
1 Concepts this Week

- Solution concepts such as NE should always include strategy profiles, and not just outcomes.
- Stackelberg model: A sequential version of the Cournot competition game.
- Extension: Dixit Model (1980 EJ).



2 Learning by Doing

1. Consider the Lansing area market for Valentine's Day Double-Date-All-You-Can-Eat Korean BBQs with the market inverse demand function $P(Q) = 510 - 5Q$ where Q is the number of tables of four customers served. There are 2 restaurants in this market, KPOT (K) and IPOT (I). The production of each table served requires one server (\$10) and one KBBQ table (\$100).

   (a) Suppose that KPOT moves first to decide how many tables to serve that day. What is the SPNE outcome and profit?

   Solution.

   The marginal cost of production is constant at \$110 a table. Since I moves second, they must maximize their profit given the quantity produced by KPOT q_K. Their maximization problem is:

   $$\max_{q_I} \, (510 - 5q_K - 5q_I - 110) \cdot q_I$$

   with the best response $q_I^* = 40 - \frac{1}{2}q_K$. So KPOT's maximization problem is:

   $$\max_{q_K} \left(510 - 5q_K - 5\left(40 - \frac{1}{2}q_K\right) - 110\right) \cdot q_K$$

   and the best response if thus $q_K^* = 40 \;\Rightarrow\; q_I^* = 20 \;\Rightarrow\; (\pi_K, \pi_I) = (4000, 2000)$

(b) Suppose now we change up the game so that KPOT is not allowed to choose their quantity first, but instead they are allowed to rent tables first. The rule of the game is as follows. Early in the morning, KPOT rents q_T tables to achieve capacity to serve q_T tables that day. By lunch time, KPOT and IPOT simultaneously choose quantities q_K and q_I to produce. KPOT only has to rent extra tables if $q_K > q_T$, but IPOT must pay rent for all q_I tables. Show that the Stackelberg outcome can still be achieved in SPNE even though the competition during and after lunch time is Cournot.

Solution.

Suppose that it is now lunch time. Notice that IPOT's maximization problem did not change, meaning their best response is still $q_I^* = 40 - \frac{1}{2}q_K$. KPOT's problem, however, is now:

$$\max_{q_K} \; [(510 - 5q_I - 5q_K) - 10 - \mathbb{1}\{q_K > q_T\} \cdot 100 \cdot (q_K - q_T)] \, q_K$$

If $q_K^* \leq q_T$, then KPOT's best response is $q_K^* = 40 - \frac{1}{2}q_I \;\Rightarrow\; (q_K^*, q_I^*) = (40, 20)$ and the profit would be $(\pi_K^*, \pi_I^*) = (8000 - 100q_T, 2000)$. In SPNE, q_T is then optimally chosen to be 40.

2. (MSU 2018, Midterm 2 Q1) It's Valentine's day! 1-800-flowers (firm 1), the Beal botanical garden (firm 2), and Costco Wholesale (firm 3) operate in the MSU market with inverse demand curve of valentine's day bouquet given by $P(Q) = a - Q$ where $a > 0$, $Q = q_1 + q_2 + q_3$, and q_i is the quantity produced by firm $i \in \{1, 2, 3\}$. Each firm has 0 cost of production. The firms choose their quantities as follows: (1) firm 1 and firm 2 choose $q_1 \geq 0$ and $q_2 \geq 0$ simultaneously; (2) firm 3 observes q_1 and q_2 and chooses $q_3 \geq 0$. What is the subgame perfect Nash equilibrium outcome of this game?

Solution.

By backward inductions, in the second stage, firm 3 takes $\bar{q}_1$ and $\bar{q}_2$ produced in first stage as given and solves:

$$\max_{q_3} \ \pi_3(q_3, \bar{q}_1, \bar{q}_2) = (a - \bar{q}_1 - \bar{q}_2 - q_3)q_3$$

The best response of firm 3 is thus

$$q_3^* = \frac{a - \bar{q}_1 - \bar{q}_2}{2}$$

In the first-stage, firms 1 and 2 simultaneously choose q_1 and q_2 by solving:

$$\max_{q_i} \left[a - q_i - q_j - \frac{1}{2}(a - q_i - q_j) \right] q_i$$

so their best responses are:

$$q_i^* = \frac{1}{2}(a - q_j) \text{ By symmetry,} q_j = \frac{1}{3}a \ \Rightarrow \ q_3^* = \frac{1}{6}a$$

In SPNE, firms 1 and 2 produce $q_1^* = q_2^* = \frac{1}{3}a$ and firm 3 produces $q_3^* = \frac{1}{3}(a - q_1 - q_2) = \frac{1}{6}a$.

3. (Modified from UPenn Prelim SS 2017 Micro II Q1) Consider a Cournot duopoly game between chocolate manufacturer *Lindt* (frim 1) and *Hershey's* (firm 2). The market price for valentine's day chocolate is given by $1-q_1-q_2$ where q_1 and q_2 are quantities of output produced by the two firms with 0 marginal costs.

   (a) Suppose that the owner of Lindt first hires a manager, after which the manager of Lindt and owner of Hershey's simultaneously choose outputs q_1 and q_2. The manager of Lindt is paid $\kappa\pi_1(q_1,q_2)+\lambda q_1-B$, where q_1 is the quantity chosen by the manager, $\pi_1(q_1,q_2)$ is the profit earned in the duopoly game (given outputs q_1 and q_2), and κ, λ, and B is a non-negative constant chosen by the owner of Lindt. The outside option for the manager is 0. Assume that Hershey's observes the values of κ, λ, and B before the two firms simultaneously choose their outputs. In the Cournot subgame, what are the NEs?

Solution.

Note that there is a distinct subgame corresponding to each choice of (κ, λ, B) by Lindt. By backward induction, we first solve for a NE of the subgame reached by a given (κ, λ, B). The manager of Lindt chooses q_1 to maximize

$$\kappa(1-q_1-q_2)q_1+\lambda q_1-B \;\Rightarrow\; \text{BR is } q_1=\frac{1+(\lambda/\kappa)-q_2}{2}.$$

Hershey's best response is $q_2=\frac{1-q_1}{2}$ (Notice that, by hiring the manager, Lindt's production is increased because of the term λ/κ). The equilibrium outcome is thus $(q_1^*,q_2^*)=\left(\frac{2(\lambda/\kappa)+1}{3},\frac{1-(\lambda/\kappa)}{3}\right)$, where if $\lambda/\kappa<1$, we have a unique NE, and if $\lambda/\kappa\geq 1$, the NE is where Hershey's will not produce and price will be non-positive (since Lindt produces $q_1\geq 1$ and will take a loss).

(b) Solve for the unique SPNE in this game. Compare the result to the outcome of the Stackelberg model (without managers).

Solution.

From (a), we know that Lindt will not make profit if $\lambda/\kappa \geq 1$. As such, we focus on the case where $\lambda/\kappa < 1$.

When hiring the manager, Lindt solves:

$$\max_{(\kappa,\lambda,B)} (1-\kappa)\pi_1(q_1^*, q_2^*) - \lambda q_1^* + B \;\Rightarrow\; \text{Need to maximize } B.$$

The highest possible B is such that the manager makes 0 when choosing q_1^* optimally, meaning $B = \kappa\pi_1(q_1^*, q_2^*) + \lambda q_1^*(\kappa, \lambda)$ and the firm's real problem is:

$$\max_{(\kappa,\lambda)} \pi_1(q_1^*(\kappa,\lambda), q_2^*(\kappa,\lambda)) \equiv \max_{(\kappa,\lambda)} \left(\frac{1-(\lambda/\kappa)}{3}\right) \frac{2(\lambda/\kappa)+1}{3}.$$

The F.O.C. with respect to (λ/κ) is $-4\frac{\lambda}{\kappa} + 1 = 0 \;\Rightarrow\; \frac{\lambda}{\kappa} = \frac{1}{4}$. Now, notice that the formulation of this profit maximization problem of Lindt is identical to that of the Stackelberg model. Specifically, Hershey's must respond to an overproduction of Lindt, and Lindt counts on that response to justify the overproduction. As such, the optimal choice of (κ, λ) will achieve the Stackelberg outcome with Lindt as the leader and $\lambda/\kappa = \frac{1}{4}$.

The unique SPNE of this game is where Lindt chooses the contract $(\kappa, \lambda, B) = (4\lambda, \lambda > 0, \kappa\pi_1(q_1, q_2) + \lambda q_1)$, the manager accepts contract if $\kappa\pi_1(q_1, q_2) + \lambda q_1 - B \geq 0$ and chooses $q_1^* = \frac{2(\lambda/\kappa)+1}{3} = \frac{1}{2}$ if they accept the contract, and Hershey's produces $q_2^* = \frac{1-(\lambda/\kappa)}{3} = \frac{1}{4}$.

(c) Now suppose that both owners hire managers, simultaneously making public the terms of each manager's contract $(\kappa_i, \lambda_i, B_i)$. Then, the managers simultaneously choose outputs. What is the unique SPNE of this game?

Solution.

From part (b), we know that only the ratio λ/κ matters. Let us define $\gamma_i = \lambda_i/\kappa_i$. From firm 1's BR in part (a), we know that the two firms have the best response functions:

$$q_i^* = \frac{1 + \gamma_i - q_2}{2}.$$

Solving this system of 2 equations, we get:

$$q_i^* = \frac{1 + 2\gamma_i - \gamma_j}{3}.$$

Firm i's profit maximization problem is thus:

$$\max_{\gamma_i} \left(1 - \frac{2 + \gamma_i + \gamma_j}{3}\right)\left(\frac{1 + 2\gamma_i - \gamma_j}{3}\right).$$

The F.O.C. is:

$$1 = 4\gamma_i + \gamma_j.$$

Solving this system we get $\gamma_i^* = \gamma_j^* = \frac{1}{5}$.

(d) How do firms' outputs and profits in the previous part compare to the NE outcomes without managers? Give an explanation using your economic intuition.

Solution.
In equilibrium, both firms subsidize output. Equilibrium output must then be higher, and hence profits lower, than in a Nash equilibrium without subsidies. The firms are effectively playing a Prisoners' Dilemma.

(e) Suppose that a law is proposed to make disclosing the compensation contracts of managers illegal. Given that the owners always have the option of not disclosing such contracts, why would such a law have any effect? Would you expect the owners of the two firms to support this law?

Solution.
Either firm would prefer to be the only firm offering an observable contract, but both fare worse when both do so. Nonetheless, in the absence of the law, it is not an equilibrium to have only one firm disclosing the contract. The firms would support such a law.

4. (MSU Prelim FS 2016 Pt II Q3) There is one upstream firm (supplier), Firm U, and one downstream firm (retailer), Firm D. In the absence of entry, the total profit of the supply chain is $\pi > 0$ and that this profit is shared equally between U and D, possibly through bargaining. Assume that a potential entrant in the upstream market emerges with probability $\alpha \in (0, 1)$ and a potential entrant in the downstream market emerges with the same probability α. Also assume these events to be independently drawn. Suppose each entrant has to pay a positive but very small but entry cost $\varepsilon > 0$.
If only upstream entry occurs, the incumbent upstream firm U earns zero profit, and the joint profit of the incumbent downstream firm D and the new upstream entrant is $\Pi > \pi$. Similarly, if only downstream entry occurs, D earns zero profit, and the joint profit of U and new downstream entrant is Π. (The assumption of $\Pi > \pi$ is to capture that the potential entrants are more efficient than the incumbents.) Assume that after entry occurs, the upstream and downstream firms still share the industry profit equally.
If entries occur in both the upstream and downstream markets, then the new entrants can partner with each other to make a positive profit. In that case, because ε is assumed to be small, both entrants will stay, and both incumbent firms will earn zero profit.

(a) Derive the expected profits of the incumbent upstream and downstream firms, U and D.

Solution.

The upstream and downstream firms' respective expected profits are

$$\pi_U = \pi_D = (1 - \alpha)\frac{1}{2}[\alpha\Pi + (1 - \alpha)\pi]$$

Now, suppose U and D can sign an exclusive contract, agreeing not to deal with any new entrant. When an upstream entrant arrives, in the absence of a downstream firm to partner with it, it will not make any profit. Since there is a positive entry cost ε, it will choose not to enter. Similarly, when an upstream entrant arrives, in the absence of a downstream firm to partner with it, it will not make any profit, and will choose not to enter. However, if there is entry in both the upstream and downstream markets, then since the new entrants can partner with each other to make a positive profit, the exclusive contract cannot deter their entries.
Summing up, the exclusive contract between U and D is ineffective in blocking simultaneous entries in both the upstream and downstream markets, but is effective in blocking entry if it occurs only in the upstream or the downstream market.

(b) Calculate the expected joint profit of U and D if they sign an exclusive contract.

Solution.
$(1-\alpha)^2\pi$

(c) Suppose U and D will sign an exclusive contact if and only if doing so leads to an increase in the expected joint profit. Derive the condition under which an exclusive contract will be signed.

Solution.
It follows that exclusive contract is jointly optimal if and only if

$$\begin{aligned}(1-\alpha^2)\pi &> \pi_U + \pi_D \\ &= (1-\alpha)[\alpha\Pi + (1-\alpha)\pi] \\ (1+\alpha)\pi &> [\alpha\Pi + (1-\alpha)\pi] \\ 2\alpha\pi &> \alpha\Pi \\ 2\pi &> \Pi\end{aligned}$$

3 Go the Extra Mile

1. (MSU Prelim SS 2011 Part I Q2) Let $c : [0, 1] \to \mathbb{R}$ and $p : [0, 1] \to \mathbb{R}$ be two functions such that $c(a) < p(a)$ for all $a \in [0, 1]$. Consider the following game played between two players "Child" (C) and "Parent" (P). Child moves first and picks $a \in [0, 1]$. Parent observes the choice of a made by C and picks a real number $T \in \mathbb{R}$. The game now ends with the payoffs $c(a)+T$ for C and $\min\{p(a)-T, c(a)+T\}$ for P. Show that in SPNE, C chooses a to maximize $c(a) + p(a)$.

2. (MSU Prelim FS 2014 Part II Q4) Consider the following game of strategic communication between a Judge and a Plaintiff. The Plaintiff has been injured. Severity of the injury, denoted by θ, is the Plaintiff's private information. The Judge does not know θ and believes that θ is uniformly distributed on $\{0, 1, 2, \dots, 99\}$ (so that the probability that $\theta = i$ is $\frac{1}{100}$ for any $i \in \{0, 1, \dots, 99\}$. The Plaintiff can verifiably reveal θ to the Judge without any cost, in which case the Judge will know θ. The order of the events is as follows. First, the Plaintiff decides whether to reveal θ or not. Then, the Judge rewards a compensation y. Hence, the strategy of the Plaintiff with injury θ is $s_P(\theta) \in \{\theta, \emptyset\}$ where $\emptyset$ denotes that case of where θ is not revealed. The strategy for the Judge is $s_J : \{0, 1, \dots, 99\} \to [0, \infty)$. The payoff of the Plaintiff is $y - \theta$, and the payoff of the Judge is $-(y - \theta)^2$. Everything described so far is common knowledge. Find a (weak) perfect Bayesian equilibrium of this game.